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AN ANALYSIS OF RESPONSE DISTRIBUTIONS FROM THREE DIFFERENT ETIOLOGICAL GROUPS OF EDUCATIONALLY HANDICAPPED LEARNERS

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Monterey, California

THESIS

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by

Wolfgang W. Oertel

September 1975

Thesis Advisor: J. K. Arima

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with different etiologies were analyzed using the Markovian learning model as a tool. The learning task was a remedial language program created by the Behavioral Sciences Institute, Carmel, California. No differences were found in the learning patterns of these groups. Rather, a positive relationship supporting the behavioral viewpoint was found to exist.

An Analysis of Response Distributions
from
Three Different Etiological Groups of Educationally Handicapped Learners

by

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Submitted in partial fulfillment of the
requirements for the degree of

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In the area of special education there are two different approaches towards training: The differential diagnosis approach states that etiological characteristics affect prognosis for training and the behavioral approach asserts that the results of training are independent of etiological factors. To obtain more evidence regarding these positions, the response distributions of three groups of language-handicapped children with different etiologies were analyzed using the Markovian learning model as a tool. The learning task was a remedial language program created by the Behavioral Sciences Institute, Carmel, California. No differences were found in the learning patterns of these groups. Rather, a positive relationship supporting the behavioral viewpoint was found to exist.

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I. INTRODUCTION

A. THE DIFFERENTIAL DIAGNOSIS APPROACH

In the area of education, and especially special education, the predominant teaching strategy is based upon the individual specialness of the child. This unique characteristic is categorized according to etiological factors which are presumed to have contributed to the development of the learning problem. Further, these etiological characteristics are presumed to affect prognosis for training as well as indicate the specific method of instruction.

One result of this type of approach to learning problems is a significant investment of time and effort in the area of differential diagnosis. One area in which this is clearly seen is in the area of language learning problems. Here, the preponderance of tests and etiological classification techniques is far greater and more highly developed than are procedures to actually teach language to nonlanguage children.

For each child who has been referred to the psychologist because of retarded functioning, Kastein [Ref. 1] requires a clinical study that takes into account the total developmental history of the child. She writes:

> In order to evaluate language and speech development, a differential diagnostic evaluation of the language and speech development of the child should be made. A thorough history is obtained from the parent, with emphasis on pre-natal, neo-natal, and post-natal development, motor development, and social maturity ... (Page 168)

Further, she believes that the child's perception of spatial relationships, visuo-motor functions, and gestalt perception are integral parts of language development. She argues that since multiple factors determine a child's communication disorder, only a team approach of various disciplines can determine all of the causative factors in the impaired function of language. This knowledge of multiple etiology is necessary in order to assure adequate training and rehabilitation.

Michal-Smith [Ref. 1] also stresses the importance of etiology as a vital factor in establishing valid therapeutic and educational techniques for the handicapped learner. Dealing with the special case of mental retardation, he states: "Although we may arrive at a general diagnosis of mental retardation, it is also necessary to establish a differential diagnosis; that is, to attempt to determine the etiologic factors in mental retardation." (Page 318)

Freburg Berry [Ref. 2] develops an educational plan based upon group instruction, not individual tutoring. She recommends different longitudinal studies, including a case history and the study of developmental factors contributing to the child's language behavior. Her suggested case history would contain information about family history, pregnancy data, perinatal history, and socioemotional development. In her guidelines she argues that it would not be sufficient only to analyze the child's language behavior, but that the crucial deficit in the child's language behavior is often discovered by observing other facets of behavior. "It may not seem important," she

writes, "to note evidences of motor awkwardness or bizarre movement patterns in avoidance of objects or in accommodation to space. It is important, however, because poor psychomotor coordination may be the first notable sign of a central neural deficit that also may have affected the child's language." (Page 191)

The views of three authors were presented above. Convinced that the only effective teaching strategy for a handicapped learner is one that is built on his individual specialness, they are typical representatives of the differential diagnosis approach.

B. THE BEHAVIORAL VIEWPOINT

There is an alternate point of view, however. Arising generally from a basic, performance-oriented approach is the philosophy that it is more important to describe the child's actual language abilities and needs than to chronicle his developmental past. Most behaviorists, however, still recognize broad etiological classifications. Arising from this general set is an unusually different position which presumes that whatever deficit in associated skills may be present, these factors do not adversely affect prognosis nor do they influence the type of training which should be used.

Reger, Schroeder, and Uschold [Ref. 3] consider the diagnostic process as "basically consisting of after-the-fact labeling because the problems always have been identified before the diagnostic sessions ever begin." (Page 15) They fear that the label applied to the child serves merely as a

sanction for some kind of administrative action, such as placement into some special program. They feel that the diagnostic process tells nothing about a child that was not already known because nothing is added to the fund of knowledge about the child. Moreover, the diagnosis provides no information about what to do with the child after placement changes are made. Moving a child from one classroom to another is obviously an administrative action, and it is not an act of understanding or explanation. The authors strongly reject the popular opinion that there is a method of teaching reading to brain-injured children, for example. They argue that there are a variety of ways, and the way each child learns is not primarily related to the actual brain injury. Accordingly, the method of teaching should be determined by the observed needs of the individual child. They attack the differential diagnostic approach sharply:

> We are saying that grouping children on the basis of medically derived disability labels has no practical utility in the schools. Children should be grouped on the basis of their educational needs, and these needs may be defined in any number of ways. The notion that simple labels, applied by high-status authorities from outside the school, should serve as a basis for grouping children is basically nothing more than a refusal to accept responsibility for making educational decisions. It is educational laziness ... (Page 19)

And:

> The history of the close association between medical concepts and special educational practices has led to the myth that if only one could find a basic cause of a child's problems everything from that point on would be easy. In medicine, the cause of a disease is usually directly related to its treatment and cure; obviously, it is assumed by analogy, the same process must be true in education. But in most instances it is not true, and in such instances,

> as well as in those instances where it is true, there is nothing that can be done that is relevant to knowledge of the basic cause. (Page 43)

Brabner [Ref. 4] observes that one must distinguish in physically disabled persons between the physical defect and the handicap--a distinction rehabilitation counselors are keenly aware of--rather than assume that the physical impairment is causing the handicap.

Gray and Ryan [Ref. 5] see the usefulness of any diagnostic battery only for the purpose of reducing the number of clinical options that are potentially available to the teacher. Therefore, any question whose answer does not reduce the clinical options is said to be not related to the task and should be replaced. They, too, argue against the assumption that no teaching can be successfully accomplished until the original cause of the problem has been determined. They point out that unless clinical options are available which are responsive to conditions such as birth weight, duration of labor, etc., it is difficult to see any relationship between the information obtained and its intended use in formulating a teaching strategy for language. They conclude:

> The functional clinical questions to be answered are (a) What does the child do (or not do) now?, and (b) What should the child be doing? Clinical strategy must answer the question of how to get from a to b. If the philosophical model of interrogation provides us with inappropriate questions and useless answers, we should look for a more appropriate set of operational procedures to help in the construction of a strategy for language training. (Page 8)

This then is a selection of arguments from the current literature on special education, the latter representing the behavioral viewpoint. This selection is representative of

the arguments of both sides, but it does not reflect the correct proportion of their adherents: The behaviorists still form a minority.

C. MORE EVIDENCE BY AN ANALYSIS OF LEARNER RESPONSE DISTRIBUTIONS

Thus, it would appear that a paradox exists between the diagnostic approach and the behavioral viewpoint. The diagnostic-etiology philosophy has a long and respected tradition to support its use and claimed validity. On the other hand, the behavioral position just described has only a brief history and is not always accepted. However, a large body of data does exist which, taken as a whole, seems to support this latter position.

If the advocates of the differential diagnosis approach are right, i.e., if a correct teaching strategy for a handicapped learner can be built upon his individual specialness only, then this means nothing but that handicapped learners learn differently. If, on the other hand, the learning process is the same for all learners--regardless of etiology--then, obviously, the behaviorist's viewpoint is justified. In that case only one difference should be observable: A difference in the rate of acquisition. But differing rates per se can hardly be a criterion for handicapped learners as, clearly, the same phenomenon is observed among nonhandicapped learners, too.

In an effort to obtain more evidence, the author thought it helpful to explore on an analytical level the two positions in education by actually testing for any differences in learning due to different etiologies.

The response distribution of learners can be described as a finite, absorbing Markov chain, where states represent the level at which an item has been processed by the learner. The Markovian model constituted a unique tool for the analysis intended. That is, by noting the parameters of a Markov model and the pattern of their changes an investigator can make relatively strong inferences about the nature of differences occurring in the learning process.

The feasibility of such an analysis was made possible as a result of the author's association with the Behavioral Sciences Institute of Carmel, California: data were readily available. Members of the Institute have developed a procedure for preparing language training programs for nonlanguage children. This procedure, "Language Training through Programmed Conditioning," is based on a mathematical structure. The total language program consists of many subprograms covering different rules of grammar. The goal of the total program is to teach a student a grammatically intact oral language. Each of the 41 subprograms consists of a series of steps that sequentially train a student until he reaches a desired objective. The total program is designed to test the student continuously to determine what subprograms he needs, when he has completed a subprogram, and when he has special problems that require stepping back (branching).

The programming procedure is used to achieve the goal of moving a subject from a specific unlearned state for a given grammatical response to a learned state for that response.

The major strategy is comprised of stimulus-response consequence, frequently referred to as operant conditioning. Often the learned state can be achieved most effectively by reducing the unlearned state to a number of small steps which are learned in a sequence.

The "Monterey Language Program" of the Behavioral Sciences Institute does exactly that and thus provides unique strings of stimulus-response events which result in a most basic level of learning data.

D. NULL HYPOTHESIS

Numerous nonlanguage children with all kinds of etiologies are involved in these programs. This, the author believed, was a good opportunity to perform the analysis intended. When the diagnostic approach to language-handicapped children is justified one would expect significant differences to show up between children's response histories, arising from a different learning process due to different etiologies. When, on the other hand, a significant positive relationship exists in the learning trends of children of different diagnostic categories, this undoubtedly would support the behavioral viewpoint.

Data from three categories of educationally-handicapped children were investigated: mentally retarded, deaf, and bilingual. By choosing those three, the necessary, clear, and sharp-cut etiological distinctions seemed to be very well established. As the first category may be comprised of different etiologies, an additional division was performed within this group, based on IQ. Accordingly, category I(a) represents

children with an IQ greater than 50, commonly referred to as educationable mentally retarded (EMR), and category I(b) represents the group with an IQ less than 50, usually called trainable mentally retarded (TMR).

Two null hypotheses were required to test the relative merit of the two theoretical viewpoints. The first hypothesis states that there is no difference in the learning patterns of handicapped children from the above-mentioned categories. Rejection of this null hypothesis would support the view of the differential-diagnosis approach. The second null hypothesis states that there is no positive relationship among the learning trends of handicapped children from the different diagnostic categories. This second null hypothesis is necessary in the event that the first null hypothesis has to be accepted. Rejection of the second null hypothesis would force acceptance of the alternative hypothesis that there is a relationship among the learning trends of handicapped children regardless of diagnostic categories. This state would support the behaviorist approach toward remedial language learning. Acceptance of both null hypotheses would imply that the learning of each child is highly individualistic and unpredictable--either because of experimental error or because of the inherent unreliability of the process.

II. EXPERIMENTAL PROCEDURE

A. DESCRIPTION OF CATEGORIES AND SUBJECTS

All subjects for this analysis were randomly selected by their teachers. Their learning data were sent to the Behavioral Sciences Institute, Carmel, where the author again randomly selected five completed program steps from each subject's data. However, it was not possible to get the desired quantity in all cases.

Category I(a) consisted of 10 educable, mentally retarded (EMR) children from Houston, Texas. There were 6 males and 4 females in this group, 6 to 13 years old. All were regularly enrolled in public schools, but in special classes. Their IQ ranged from 60 to 70. One child was observed over four program steps, all others over five.

Category I(b) was the trainable mentally retarded (TMR) group. Each of 5 males and 5 females, 7 to 13 years old, were observed over five program steps. The data came from Bridgeport, Connecticut. Each subject's IQ was below 50; diagnosis was in all cases brain damage or Down's syndrome. All of these children will probably need custodial care all of their lives.

Category II consisted of 10 deaf children, 6 males and 4 females, from Fairfield, California. Their ages were 7 to 10 years. All of these children used total communication (oral with manual signs including morphological endings). In this group only one child could be observed over five program steps.

For the others, the available data allowed only an observation over two or three program steps.

Category III was the bilingual group consisting of 10 Mexican children, 7 males and 3 females, from Castroville, California. They were 7 to 11 years old and had to learn English as a second language. All children in this group could be observed over five program steps.

B. THE LEARNING PROCESS MODELLED AS A MARKOV CHAIN

The intention to use the Markovian model as an analytical tool necessitates further development of its theoretical principles. According to Atkinson, Bower, and Crothers [Ref. 6], the simplest representation of a learning process as a Markov chain would involve just two states, an unlearned state (U) and a learned state (L). Once a subject has moved to L, he will always respond correctly. As long as he is in U there is a certain probability, p, of a correct response to the stimulus. This probability should be close to chance level. Further, on each trial in U there is a probability, c, of moving into L. The transfer to L can occur only when a subject makes an error while in U and as a result selects the relevant cue to solve the task. The subject will try many cues, make errors, and finally select the correct cue necessary to solve the task. Once the relevant cue has been selected, the probability of succeeding correct responses is 1.0; that is, the subject has arrived at the absorbing state L.

Based on this theory, Coombs, Dawes, and Twersky [Ref. 7] derive the following structure of the Markov chain:

$$\begin{array}{cccc} & & \multicolumn{2}{c}{\text{trial } n+1} \\ & & L & U \\ \text{trial } n & L & 1 & 0 \\ & U & c & 1-c \end{array} \qquad \begin{array}{c} \Pr\ (\text{correct}) \\ 1 \\ p \end{array}$$

The left matrix shows the transition probabilities between states from trial to trial, and the matrix on the right is a probability vector for correct response over the conditioning states on trial n. If on trial n the subject is in state L, he will, on all subsequent trials, stay in state L, and the individual will respond correctly. If the item is in state U, it will become conditioned to the correct response, i.e., move to state L for the next trial, with probability c, or otherwise remain unconditioned, and the probability of the individual responding correctly is p. L is an absorbing state because it cannot be left, and it can be reached from state U.

There are two major assumptions in the model: The first is that successive presolution-trials, i.e., the responses in U, are statistically independent, and the second one is that presolution responses are stationary.

The first assumption of independence means that the probability of a success on any presolution-trial is the same regardless of what occurred on the preceding trial. Thus, programmed learning tasks with binary decisions are considered as independent Bernoulli trials in this model.

It is helpful to define a random variable x_n that represents the response of a particular subject on an arbitrary trial n of the experiment:

$$x_n = \begin{cases} 1, \text{ if a subject makes an error on trial } n \\ 0, \text{ if a subject is correct on trial } n \end{cases}$$

Each subject generates a particular trial sequence of values of the random variables $x_1, x_2, \ldots, x_n$. To test for independence of presolution trials, the conditional probabilities, $Pr(X_{n+1} = 0 | X_n = 0)$ and $Pr(X_{n+1} = 0 | X_n = 1)$, must be compared. If trials are independent, the conditional probabilities above are expected to be equal. This can be tested by means of a Chi-square test.

The assumption of stationarity says, basically, that subjects do not display any change in their performance over the series of presolution trials. The probability of an error on any trial before the last error remains a constant, so there should be no evidence of learning in the presolution data. This means that over any block of n trials prior to the last error the distribution of the number of correct responses made by the subjects is given by the binomial $(p + q)^n$, where p is the guessing parameter that can be estimated from the data and $q = 1 - p$. A simple test which is sensitive to the trends within each individual's response sequence is to compare the number of successes in the first and second halves of each subject's series of presolution trials. If each individual is not improving over his trials, then the number of successes in the two halves should be equal, except for sampling variability. A _t_ test may be performed on the difference scores between the first and second half in order to determine whether the data are stationary.

If a process is both independent and stationary, then it has the property that the probability of a given state is independent, both of the trial number and the preceding states.

C. THE MARKOV MODEL AS A TOOL

Atkinson, Bower and Crothers [Ref. 6] use the Markov model to derive predictions regarding statistics obtained from the observable response sequences. In an effort to test how well these predictions fit the actual data, most of the statistics used in this analysis have been evaluated in two ways: (1) by using the prediction formulas and (2) by using the data directly.

The number of sequential statistics that can be tabulated and predicted by the model are practically unlimited. The most important characteristics of the model, however, are stationarity and independence and the distribution of total errors. Atkinson, et al. [Ref. 6] showed that if these statistics conform to the predictions of the model, then in practice the remaining statistics accord well with predictions.

The more relevant statistics used in performing the analysis are defined below:

p = overall probability of a correct response. (If the subject starts the trial in U, the probability of a correct response is p)

q = overall probability of an incorrect response

$\overline{T}$ = mean number of errors

σ_T = standard deviation of T

c = rate of learning (probability of going from U to L)

σ_c = standard deviation of c

$\bar{L}$ = mean number of trials on which last error occurred

σ_L = standard deviation of $\bar{L}$

$\bar{H}$ = mean number of successes intervening between two adjacent errors

σ_H = standard deviation of $\bar{H}$.

$Pr(X_{n+1} = 0 | X_n = 1)$ = The probability of a success conditional upon the occurrence of an error on the previous trial.

$Pr(X_{n+1} = 0 | X_n = 0)$ = The probability of a success conditional upon the occurrence of a success on the previous trial.

Test for independence = The independence of presolution trials was tested by means of a Chi-square test.

Test for stationarity = Stationarity was tested by means of a _t_ test performed on the difference scores between the first and second halves of presolution trials.

These statistics are tabulated in Tables I and II. The computational formulas are shown in Appendix A.

The analysis was performed in the following order:

1) Tests for independence of presolution trials in each category
2) Tests for stationarity of presolution trials in each category
3) Computation of sequential statistics for each category
4) Tests for differences between categories by means of direct comparison and analysis of variance
5) Based on 4), rejection or acceptance of the first null hypothesis.
6) Correlation of model predictions and actual outcomes in each category
7) Based on 6), rejection or acceptance of the second null hypothesis.

III. RESULTS

A. ACTUAL AND PREDICTED LEARNING OF CATEGORICAL GROUPS

The raw data from all subjects are located in Appendix B. These data represent each subject's sequences of responses, correct (represented by the symbol 0) and incorrect (represented by the symbol 1). The "solution criterion" was 20 consecutive correct responses. That is, whenever a subject generated 20 correct responses in succession, he was assumed to have moved to the learned state L. The last 1 in each string represents the last error that occurred before reaching L. In the data, the solution state responses have been deleted, since they are a string of 0's.

Statistics describing the learning performance of each of the categorical groups are shown in Tables I and II. Table I presents the basic descriptive statistics, while Table II presents those statistical parameters that are descriptive of the empirical results and are also predictable from the Markov model. The computations are shown in Appendix A. As explained previously, each group had 10 subjects, and the number of program steps that the data are based on were 49 and 29 for groups I(a) and II, respectively. The program steps were 50 for the other two groups.

TABLE I

DESCRIPTIVE PARAMETERS OF THE PERFORMANCE OF CATEGORICAL GROUPS

Group	Parameters				
	$\hat{p}$	$P(X_{n+1}=0 \mid X_n=1)$	$P(X_{n+1}=0 \mid X_n=0)$	$\bar{T}$	$\hat{c}$
I(a) EMR	.798	.719	.817	4.96	.202
I(b) TMR	.773	.715	.789	8.00	.125
II Deaf	.781	.726	.795	4.79	.209
III Bilingual	.777	.671	.806	6.04	.165

TABLE II

EMPIRICAL AND PREDICTED PARAMETERS OF THE PERFORMANCE OF CATEGORICAL GROUPS

Group	Parameters					
	$\bar{L}$	$\bar{H}$	σ_T	σ_c	σ_L	σ_H
I(a) EMR						
Actual	24.51	5.75	5.02	.329	20.30	3.10
Predicted	24.51	3.95	4.43	.026	24.00	4.22
I(b) TMR						
Actual	35.74	4.95	7.18	.317	31.39	2.98
Predicted	35.24	3.41	7.48	.017	34.74	3.87
II Deaf						
Actual	21.79	4.68	3.89	.38	19.74	2.91
Predicted	21.89	3.57	4.26	.035	21.38	4.04
III Bilingual						
Actual	27.02	5.22	7.03	.297	26.02	3.91
Predicted	27.18	3.48	5.52	.021	26.68	3.95

B. INDEPENDENCE AND STATIONARITY

As stated, two crucial tests of learning performance with respect to the Markov model are those for independence and stationarity, since other aspects of performance are closely related to these characteristics. Independence was tested by calculating for each subject in each group the observed frequency of the four possible sequences (1,1; 1,0; 0,1; 0,0) and then computing the Chi-square values by means of the appropriate formula for a 2 x 2 contingency table (incorporating the correction for continuity). Whenever subjects had cell entries with expected frequencies of less than 5, the data were combined with as many adjacent subjects as necessary to establish a frequency of at least 5. Then the obtained Chi-square values within each group were summed and the degrees of freedom determined according to the number of independent statistics added.

The obtained values of Chi-square were 12.38 for group 1(a), 12.99 for group I(b), 6.02 for group II, and 19.04 for group III. At the 5% level, Chi-square values of 12.6 (df = 6), 15.5 (df = 8), 11.1 (df = 5), and 15.5 (df = 8) are critical for the groups, respectively (all computations are in Appendix A).

The value of 19.04 from group III is significant. However, it is 1 program-step (from subject 9) out of 50 that contributes an amount of 10.66 to the total value of 19.04. Obviously, this program-step is an outlier and has to be removed from the analysis in order to avoid biasing the data: without this particular step the obtained Chi-square value for group III

is 8.37 which is not significant. Accordingly, it was concluded that the performance of all groups was consistent with the independence hypothesis.

Stationarity was tested by comparing the proportion of correct responses in the first and second halves of the presolution trials for each group. These proportions are shown in Table III. The difference in these proportions for each group was tested by a direct-difference _t_ test, and the obtained _t_ statistics, as well as their associated degrees of freedom, are also shown in Table III. None of the obtained _t_ values were significant at the .05 level using a two-sided test. (Computations are in Appendix A).

TABLE III

PROPORTIONS OF CORRECT RESPONSES AND _t_ VALUES FOR STATIONARITY OF CATEGORICAL GROUPS

Group	Proportion correct			
	1st half	2nd half	df	t
I(a) EMR	458/565	483/565	48	1.82
I(b) TMR	665/848	688/848	49	1.32
II Deaf	233/293	243/293	28	.96
III Bilingual	502/640	529/640	49	1.54

C. TEST FOR DIFFERENCES

To test further for any differences among groups, analyses of variance were performed for $\hat{p}$ and $\bar{T}$. These are shown in Table IV and Table V, respectively. The parameters $\hat{p}$ and $\bar{T}$ were chosen as they are basic parameters on which other statistics depend. For both, $\hat{p}$ and $\bar{T}$, the null hypotheses were

that there are no differences among group means. The analyses were performed at the 5% level of significance. Accordingly, the critical F- value is $F_{.95}(3,36) = 2.89$. The chosen level of significance did not allow the rejection of one of the null hypotheses. (Computations are shown in Appendix A.)

TABLE IV

ANALYSIS OF VARIANCE OF $\hat{p}$

Source	df	SS	MS	F
Groups	3	.026	.00867	1.045
Residual	36	.299	.0083	
Total	39	.325		

TABLE V

ANALYSIS OF VARIANCE OF $\overline{T}$

Source	df	SS	MS	F
Groups	3	63.15	21.05	2.35
Residual	36	322.72	8.96	
Total	39	385.87		

D. CORRELATION BETWEEN PREDICTION AND ACTUAL OUTCOME

As a basis for establishing a positive relationship among categorical groups, correlation coefficients between model prediction and actual data outcome were determined within each group for parameters $\overline{L}$ and $\overline{H}$. Besides the standard deviations, these are the only descriptive parameters of the empirical results that were also predicted from the Markov model.

In each case, the null hypothesis was tested that there is no correlation between predicted and actual data by means of a Pearson product-moment correlation calculated for each group. The critical value of the product-moment correlation for 9 df at the 5% level of significance is .521. The correlation coefficients are shown in Table VI. (Computations are in Appendix A.) In all cases, the null hypotheses of no correlation had to be rejected.

TABLE VI

CORRELATION COEFFICIENTS
BETWEEN
MODEL PREDICTION AND ACTUAL OUTCOME

	Groups			
Parameter	I(a)	I(b)	II	III
$\overline{L}$	.9988	.9965	.9914	.9995
$\overline{H}$	.5245	.6916	.8012	.7804

IV. DISCUSSION

The performance of each categorical group, in fact, proved to be independent and stationary after removing one highly significant program step from the data of group III. There are different possible reasons that may have led to this unusual response sequence. One could be that the subject in that particular session responded obstinately to a real or imagined defect in teacher behavior. (In a pilot study the author found that error reinforcement, for example, has those effects on learners.)

According to the Markov model, independence and stationarity of presolution trials are critical and sensitive indicators of occurring learning patterns. Based on the observed high correspondence of these indicators among categories and the associated analyses of variance of $\hat{p}$ and $\overline{T}$, the first null hypothesis of no differences among categories had to be accepted.

The sequential statistics listed in Tables I and II well illustrate and further justify this decision. For example, comparing the values of p, $Pr(X_{n+1} = 0 | X_n = 1)$, $Pr(X_{n+1} = 0 | X_n = 0)$, $\overline{H}$, and σ_H over the different categories the high extent of agreement becomes clearly apparent. Differences among the categories in the rate of learning ($\hat{c}$) and in rate-related statistics, such as the mean number of errors ($\overline{T}$) and the mean number of trials to last error ($\overline{L}$) showed up as expected but in category I(b), only, which was included in the analysis as a low extreme on the IQ scale.

The analysis of variance of $\overline{T}$ proved that, although expected to do so, even the mean number of errors did not vary so much as to result in a significant F-statistic at the 5% level.

The F-statistic in the analysis of variance of $\hat{p}$ would not be significant even at a 30% level of significance which is a strong argument for the validity of the Markov model: If $\hat{p}$ was different within or between groups, the assumption that it is a guessing parameter in independent Bernoulli trials would not hold.

The acceptance of the first null hypothesis necessitated further analysis to test the second null hypothesis stating that there is no positive relationship among the learning trends of children from the different groups. One way to test for such a positive relationship was to determine for each category the correlation between model prediction and actual data outcome. For both $\overline{L}$ and $\overline{H}$, the hypotheses of no correlation had to be rejected in all groups. The relationship between predicted and actual results accounted for about 70% of the variance in $\overline{H}$ and 99% of the variance in $\overline{L}$ over all the groups. This is a good fit of the data, forcing the rejection of the second null hypothesis. In the case of $\overline{L}$, a contrary decision could only be based on a residual of less than 1% of the variance. Thus, as each treatment category learns according to the same model, it can be concluded that they learned in the same manner.

V. CONCLUSIONS

The findings strongly suggest that the learning process is the same for all subjects involved in the analysis. Undoubtedly, the results support the behaviorist approach toward remedial language learning.

Though the study concentrated on language learning exclusively, the uniformity of the findings suggests that the results may hold for other learning subjects as well.

Differences in learning rates are not restricted to etiological categories of handicapped learners; rather, they are an individual characteristic that can be found among all forms of organic life. Therefore, special classes based on rate would be more meaningful than those based on etiology.

Etiological characteristics did not affect human learning patterns in this study. The possibility remains, however, that other etiological characteristics and other learning tasks may show an etiological effect on learning. Such effects must be demonstrated empirically, however, if they are to escape the realm of conjecture.

APPENDIX A

COMPUTATIONS AND TESTS

I. COMPUTATIONAL FORMULAS FOR TABLES I AND II

The statistics listed in Tables I and II were computed according to the formulas listed below.

STATISTIC	MODEL PREDICTION	ACTUAL
$\hat{p}$	--	$P[X_{n+1}=0]$*
$\hat{q}$	--	$1 - \hat{p}$
$P(X_{n+1}=0 \mid X_n=1)$	--	*
$P(X_{n+1}=0 \mid X_n=0)$	--	*
Independence	--	*
Stationarity	--	*
$\overline{T}$	--	$\sum_{i=1}^{s} T_i/s$
$\hat{c}$	--	$1/\overline{T}$
$\overline{L}$	$1/qc$	$\sum_{i=1}^{s} L_i/s$
$\overline{H}$	$(1-q)/q$	$\sum_{i=1}^{s} H_i/s$, where $H_i = \frac{L_i - T_i}{\#\text{ correct runs}}$
σ_T	$E[T] \cdot \sqrt{1 - c}$	$\sqrt{[\Sigma T_i^2 - \frac{(\Sigma T_i)^2}{s}]/(s - 1)}$
σ_c	$\sqrt{[c^2(1-c)]/s}$	$\sqrt{[\Sigma c_i^2 - \frac{(\Sigma c_i)^2}{s}]/(s - 1)}$
σ_L	$E[L] \sqrt{1 - qc}$	$\sqrt{[\Sigma L_i^2 - \frac{(\Sigma L_i)^2}{s}]/(s - 1)}$
σ_H	$\sqrt{(1 - q)/q^2}$	$\sqrt{[\Sigma H_i^2 - \frac{(\Sigma H_i)^2}{s}]/(s - 1)}$

* ≡ See computation on next page.

s ≡ Total number of program steps.

All formulas for model prediction from Ref. 6.

II. TESTS FOR INDEPENDENCE AND STATIONARITY

Category I(a):

Test for independence

H_0: $\Pr(X_{n+1}=0|X_n=0) = \Pr(X_{n+1}=0|X_n=1)$

H_1: $\Pr(X_{n+1}=0|X_n=0) \neq \Pr(X_{n+1}=0|X_n=1)$

Let $\alpha = .05$

$$\chi^2_{(1)} = \frac{(|AD - BC| - N/2)^2 \; N}{(A+B)(C+D)(A+C)(B+D)}$$

SUBJECT	SEQUENCE 11	10	01	00	χ^2
1	11	23	24	92	1.4005
2	5	13	14	99	1.8539
3 + 4	10	28	30	192	3.16
5	6	14	14	77	1.48
6	8	31	32	74	.896
7 + 8 + 9 + 10	25	58	61	249	3.59
TOTALS	65	167	175	783	12.38

$$\chi^2_{df = 6 \text{ at } .05} = 12.6$$

As 12.38 < 12.6, accept H_0.

$\Pr(X_{n+1}=0|X_n=0) = \frac{783}{958} = .817$

$\Pr(X_{n+1}=0|X_n=1) = \frac{167}{232} = .719$

$\hat{p} = \Pr(X_{n+1}=0) = \frac{950}{1190} = .798$

Category I(a):

Test for stationarity

Each subject's presolution trials were divided into two halves and the proportion of correct responses in both halves was determined (last error excluded). A t-test was run on the difference scores.

Total proportion correct first half	Total proportion correct second half	Total difference
458/565	483/565	+25

H_0: $\mu = 0$

H_1: $\mu \neq 0$

Let $\alpha = .05$

$$t = \frac{(\overline{X} - \mu_0)}{\sigma / \sqrt{S}}, \text{where } \sigma = \sqrt{\frac{\Sigma X_i^2 - (\Sigma X_i)^2 / S}{S - 1}}$$

$\overline{X} = \frac{25}{49} = .51$, $\mu_0 = 0$, $\sigma = 1.959$

$t = 1.822$

$t_{df=48, \text{ at } .025} = 2.01$

Rejection region: $t > 2.01$ or $t < -2.01$

Thus, accept H_0.

Category I(b):

Test for independence

$H_0 = \Pr(X_{n+1}=0|X_n=0) = \Pr(X_{n+1}=0|X_n=1)$

$H_1 = \Pr(X_{n+1}=0|X_n=0) \neq \Pr(X_{n+1}=0|X_n=1)$

Let $\alpha = .05$

$$\chi^2_{(1)} = \frac{(|AD - BC| - N/2)^2 \; N}{(A+B)(C+D)(A+C)(B+D)}$$

	SEQUENCE				
SUBJECT	11	10	01	00	χ^2
1	27	46	46	104	.6267
2	4	27	28	109	.5062
3	10	45	45	155	.2544
4	19	31	32	83	1.2468
5 + 6 + 7	14	59	62	253	.0043
8	21	29	30	104	6.0459
9	6	12	12	50	.8643
10	10	30	31	217	3.4436
TOTALS	111	279	286	1075	12.992

$$\chi^2_{df = 8 \text{ at } .05} = 15.5$$

As $12.992 < 15.5$, accept H_0

$$\Pr(X_{n+1}=0|X_n=0) = \frac{1075}{1361} = .789$$

$$\Pr(X_{n+1}=0|X_n=1) = \frac{279}{390} = .715$$

$$\hat{p} = \Pr(X_{n+1}=0) = \frac{1354}{1751} = .773$$

Category I(b):

Test for stationarity

Procedure: See category I(a), test for stationarity.

Total proportion correct first half	Total proportion correct second half	Total difference
665/848	688/848	+23

$H_0: \mu = 0$

$H_1: \mu \neq 0$

Let $\alpha = .05$

$$t = \frac{(\bar{X} - \mu_0)}{\sigma/\sqrt{S}}, \text{ where } \sigma = \sqrt{\frac{\Sigma X_i^2 - (\Sigma X_i)^2/S}{S - 1}}$$

$\bar{X} = \frac{23}{50} = .46$, $\mu_0 = 0$, $\sigma = 2.468$

$t = 1.32$

$t_{df=49,\ at\ .025} = 2.01$

Rejection region: $t > 2.01$ or $t < -2.01$

Thus, accept H_0.

Category II:

Test for independence

H_0: $Pr(X_{n+1}=0|X_n=0) = Pr(X_{n+1}=0|X_n=1)$

H_1: $Pr(X_{n+1}=0|X_n=0) \neq Pr(X_{n+1}=0|X_n=1)$

Let $\alpha = .05$

$$\chi^2_{(1)} = \frac{(|AD - BC| - N/2)^2 \; N}{(A+B)(C+D)(A+C)(B+D)}$$

	SEQUENCE				
SUBJECT	11	10	01	00	χ^2
1	7	12	12	30	.12072
2 + 3	6	27	29	74	.83126
4 + 5 + 6	7	19	21	112	1.1699
7 + 8	9	12	14	60	3.8872
9 + 10	6	23	25	116	.01250
TOTALS	35	93	101	392	6.02158

$$\chi^2_{df = 5 \text{ at } .05} = 11.1$$

As 6.02 < 11.1, accept H_0

$$Pr(X_{n+1}=0|X_n=0) = \frac{392}{493} = .795$$

$$Pr(X_{n+1}=0|X_n=1) = \frac{93}{128} = .726$$

$$\hat{p} = Pr(X_{n+1}=0) = \frac{485}{621} = .781$$

Category II:

Test for stationarity

Procedure: See category I(a), test for stationarity.

Total proportion correct first half	Total proportion correct second half	Total difference
233/293	243/293	+10

H_0: $\mu = 0$

H_1: $\mu \neq 0$

Let $\alpha = .05$

$$t = \frac{(\overline{X} - \mu_o)}{\sigma/\sqrt{S}}, \text{ where } \sigma = \sqrt{\frac{\Sigma X_i^2 - (\Sigma X_i)^2/S}{S - 1}}$$

$\overline{X} = \frac{10}{29} = .3448$, $\mu_o = 0$, $\sigma = 1.9323$

$t = .961$

$t_{df=28, \text{ at } .025} = 2.045$

Rejection region: $t > 2.045$ or $t < -2.045$

Thus, accept H_0.

Category III:

Test for indepencence

H_0: $Pr(X_{n+1}=0|X_n=0) = Pr(X_{n+1}=0|X_n=1)$

H_1: $Pr(X_{n+1}=0|X_n=0) \neq Pr(X_{n+1}=0|X_n=1)$

Let $\alpha = .05$

$$\chi^2_{(1)} = \frac{(|AD - BC| - N/2)^2 N}{(A+B)(C+D)(A+C)(B+D)}$$

SUBJECT	SEQUENCE 11	10	01	00	χ^2
1 + 2	13	37	39	157	.56142
3	12	14	15	19	.0109
4	11	15	16	43	1.2840
5 + 6	6	31	33	168	.000008
7	11	21	21	94	2.9294
8	8	25	25	123	.5470
9	23	37	37	175	10.6756
10	12	16	17	57	3.0304
TOTALS	96	196	203	846	19.0387

$$\chi^2_{df=8 \text{ at } .05} = 15.5$$

As 19.0387 > 15.5, reject H_0

$$Pr(X_{n+1}=0|X_n=0) = \frac{846}{1049} = .806$$

$$Pr(X_{n+1}=0|X_n=1) = \frac{196}{292} = .671$$

$$\hat{p} = Pr(X_{n+1}=0) = \frac{1042}{1341} = .777$$

Category III:

Test for stationarity

Procedure: See category I(a), test for stationarity.

Total proportion correct first half	Total proportion correct second half	Total difference
502/640	529/640	+27

H_0: $\mu = 0$

H_1: $\mu \neq 0$

Let $\alpha = .05$

$$t = \frac{(\overline{X} - \mu_o)}{\sigma/\sqrt{S}}, \text{ where } \sigma = \sqrt{\frac{\Sigma X_i - (\Sigma X_i)^2/S}{S - 1}}$$

$\overline{X} = \frac{27}{50} = .54$, $\mu_o = 0$, $\sigma = 2.476$

$t = 1.54$

$t_{df=49 \text{ at } .025} = 2.01$

Rejection region: $t > 2.01$ or $t < -2.01$

Thus, accept H_0.

III. COMPUTATIONS FOR TABLES IV AND V

1) Computations for ANOVA of $\hat{p}$

	GROUP			
SUBJECT	I(a)	I(b)	II	III
1	.766	.672	.688	.819
2	.706	.809	.677	.788
3	.854	.784	.762	.550
4	.852	.691	.632	.682
5	.819	.777	.869	.836
6	.838	.836	.831	.836
7	.724	.853	.429	.782
8	.878	.722	.815	.818
9	.825	.755	.798	.779
10	.784	.857	.837	.716
	8.046	7.756	7.338	7.606

$$SS_{total} = (.766)^2 + (.706)^2 + \ldots + (.779)^2 + (.716)^2 - \frac{(30.746)^2}{40}$$

$$= .325$$

$$SS_{group} = \frac{(8.046)^2 + (7.756)^2 + (7.338)^2 + (7.606)^2}{10} - \frac{(30.746)^2}{40}$$

$$= .026$$

$$SS_{res} = SS_{total} - SS_{group}$$

$$= .325 - (.026)$$

$$= .299$$

2) Computations for ANOVA of $\bar{T}$

	GROUP			
SUBJECT	I(a)	I(b)	II	III
1	7.0	14.8	4.0	2.6
2	10.8	6.4	5.0	7.8
3	3.8	11.2	8.33	5.4
4	4.6	10.2	2.66	5.4
5	4.2	9.8	3.0	3.8
6	3.4	4.4	4.33	4.0
7	8.0	1.0	4.0	6.6
8	2.4	10.2	5.0	6.8
9	2.8	3.8	8.5	12.2
10	2.0	8.2	4.66	5.8
	49.0	80.0	49.48	60.4

$$SS_{total} = (7.0)^2 + (10.8)^2 + \ldots + (5.8)^2 - \frac{(238.88)^2}{40}$$

$$= 385.87$$

$$SS_{group} = \frac{(49.0)^2 + (80)^2 + (49.48)^2 + (60.4)^2}{10} - \frac{(238.88)^2}{40}$$

$$= 63.15$$

$$SS_{res} = SS_{total} - SS_{group}$$

$$= 385.87 - 63.15$$

$$= 322.72$$

IV. COMPUTATIONS FOR TABLE VI

Correlation for L

Category I(a)

Let X ≡ actual data and Y ≡ model prediction

Y- values were computed according to $L = \frac{1}{q-c}$

SUBJECT	q	c	X	Y
1	.234	.143	30.2	29.88
2	.146	.263	26.4	26.04
3	.148	.217	31.2	31.14
4	.162	.294	21.0	21.00
5	.181	.238	22.4	23.21
6	.276	.125	29.2	28.99
7	.294	.093	36.0	36.57
8	.122	.417	20.0	19.66
9	.175	.357	16.2	16.01
10	.216	.500	9.5	9.26

H_0: $r_{XY} = 0$

H_1: $r_{XY} > 0$

Let $\alpha = .05$

$$r_{XY} = \frac{n\,\Sigma XY - (\Sigma X)(\Sigma Y)}{\sqrt{n\Sigma X^2 - (\Sigma X)^2}\;\sqrt{n\Sigma Y^2 - (\Sigma Y)^2}}$$

$$= \frac{10(6425.864) - (242.1)(241.76)}{\sqrt{10(6426.53) - (242.1)^2}\;\sqrt{10(6426.67) - (241.76)^2}}$$

$$= .99883$$

$r_{XY,\,df=9,\text{ at }.05} = .521$

As $.9988 > .521$, reject H_0.

Correlation for H

Category I(a)

Let X ≡ actual data and Y ≡ model prediction

Y- values were computed according to $H = \frac{1-q}{q}$

SUBJECT	q	1-q	X	Y
1	.234	.766	5.44	3.27
2	.146	.854	7.03	5.85
3	.148	.852	8.30	5.76
4	.162	.838	4.67	5.17
5	.181	.819	6.56	4.52
6	.276	.724	2.95	2.62
7	.294	.706	6.41	2.40
8	.122	.878	6.22	7.20
9	.175	.825	5.73	4.71
10	.216	.784	4.00	3.63

H_0: $r_{XY} = 0$

H_1: $r_{XY} > 0$

Let $\alpha = .05$

$$r_{XY} = \frac{n\,\Sigma XY - (\Sigma X)(\Sigma Y)}{\sqrt{n\Sigma(X^2) - (\Sigma X)^2}\ \sqrt{n\Sigma Y^2 - (\Sigma Y)^2}}$$

$$r_{XY} = \frac{10(269.9227) - (57.31)(45.13)}{\sqrt{10(350.059) - (57.31)^2}\ \sqrt{10(225.0777) - (45.13)^2}}$$

$$= .5245$$

$r_{XY,df = 9,\ at\ .05} = .521$

As $.5245 > .521$, reject H_0.

Correlation for L

Category I(b):

Let $X \equiv$ actual data and $Y \equiv$ model prediction

Y- values were computed according to $L = \frac{1}{q \cdot c}$

SUBJECT	q	c	X	Y
1	.328	.067	49.6	45.50
2	.191	.156	33.8	33.56
3	.216	.089	51.2	52.02
4	.309	.098	33.8	33.02
5	.223	.102	44	43.96
6	.164	.227	27	26.86
7	.147	1.0	7	6.80
8	.278	.098	37	36.71
9	.225	.263	16.2	16.89
10	.143	.122	57.8	57.32

H_0: $r_{XY} = 0$

H_1: $r_{XY} > 0$

Let $\alpha = .05$

$$r_{XY} = \frac{n\Sigma XY - (\Sigma X)(\Sigma Y)}{\sqrt{n\Sigma X^2 - (\Sigma X)^2}\ \sqrt{n\Sigma Y^2 - (\Sigma Y)^2}}$$

$$= \frac{10(14822.67) - (357.4)(352.64)}{\sqrt{10(15052.76) - (357.4)^2}\ \sqrt{10(14611.58) - (352.64)^2}}$$

$$= .9965$$

$r_{XY,\text{df}=9,\text{ at }.05} = .521$

As $.9965 > .521$, reject H_0.

Correlation for H

Category I(b):

Let X ≡ actual data and Y ≡ model prediction

Y- values were computed according to $H = \frac{1-q}{q}$

SUBJECT	q	1-q	X	Y
1	.328	.672	4.68	2.05
2	.191	.809	4.80	4.24
3	.216	.784	4.78	3.63
4	.309	.691	4.22	2.24
5	.223	.777	4.57	3.48
6	.164	.836	4.41	5.09
7	.147	.853	6.00	5.80
8	.278	.722	3.94	2.59
9	.225	.775	5.78	3.44
10	.143	.857	6.32	5.99

H_0: $r_{XY} = 0$

H_1: $r_{XY} > 0$

Let = .05

$$r_{XY} = \frac{n\Sigma XY - (\Sigma X)(\Sigma Y)}{\sqrt{n\Sigma(X^2) - (\Sigma X)^2}\ \sqrt{n\Sigma Y^2 - (\Sigma Y)^2}}$$

$$r_{XY} = \frac{10(197.85) - (49.5)(38.55)}{\sqrt{10(250.81) - (49.5)^2}\ \sqrt{10(166.46) - (38.55)^2}}$$

$$= .69156$$

$r_{XY,df=9,\ at\ .05} = .521$

As .69156 > .521, reject H_0.

Correlation for L

Category II

Let $X \equiv$ actual data and $Y \equiv$ model prediction

Y values were computed according to $L = \frac{1}{q \cdot c}$

SUBJECT	q	c	X	Y
1	.312	.250	12.40	12.82
2	.323	.200	16.00	15.48
3	.238	.120	35.3	35.01
4	.368	.375	6.66	7.25
5	.131	.333	23.33	22.92
6	.169	.230	24.33	25.73
7	.571	.250	7.50	7.01
8	.185	.200	27.33	27.03
9	.202	.117	42.50	42.31
10	.163	.182	29.00	33.70

H_0: $r_{XY} = 0$

H_1: $r_{XY} > 0$

Let $\alpha = .05$

$$r_{XY} = \frac{n\Sigma XY - (\Sigma X)(\Sigma Y)}{\sqrt{n\Sigma X^2 - (\Sigma X)^2}\ \sqrt{n\Sigma Y^2 - (\Sigma Y)^2}}$$

$$r_{XY} = \frac{10(6418.3) - (224.35)(229.26)}{\sqrt{10(6286.87) - (224.35)^2}\ \sqrt{10(6575.19) - (229.26)^2}}$$

$$= .9914$$

$r_{XY,df=9,\ at\ .05} = .521$

As $.9914 > .521$, reject H_0.

Correlation for H

Category II

Let X ≡ actual data and Y ≡ model prediction

Y values were computed according to $H = \frac{1-q}{q}$

SUBJECT	q	1-q	X	Y
1	.312	.688	2.63	2.21
2	.323	.677	2.69	2.10
3	.238	.762	3.93	3.20
4	.368	.632	2.17	1.72
5	.131	.869	7.39	6.63
6	.169	.831	8.87	4.92
7	.571	.429	3.5	.75
8	.185	.815	6.65	4.41
9	.202	.798	4.67	3.95
10	.163	.837	4.62	5.14

H_0: $r_{XY} = 0$

H_1: $r_{XY} > 0$

Let $\alpha = .05$

$$r_{XY} = \frac{n\Sigma XY - (\Sigma X)(\Sigma Y)}{\sqrt{n\Sigma X^2 - (\Sigma X)^2}\ \sqrt{n\Sigma Y^2 - (\Sigma Y)^2}}$$

$$r_{XY} = \frac{10(194.55) - (47.12)(35.03)}{\sqrt{10(267.22) - (47.12)^2}\ \sqrt{10(152.69) - (35.03)^2}}$$

$$= .80115$$

$r_{XY,df=9,\text{ at }.05} = .521$

As $.80115 > .521$, reject H_0.

Correlation for L

Category III

Let X ≡ actual data and Y ≡ model prediction

Y values were computed according to $L = \frac{1}{q \cdot c}$

SUBJECT	q	c	X	Y
1	.181	.385	14.40	14.35
2	.212	.128	37.00	36.85
3	.450	.185	12.20	12.01
4	.318	.185	17.20	16.99
5	.164	.263	23.40	23.18
6	.164	.250	25.00	24.39
7	.218	.151	29.60	30.38
8	.182	.147	36.40	37.38
9	.221	.082	54.60	55.18
10	.284	.172	20.60	20.47

H_0: $r_{XY} = 0$

H_1: $r_{XY} > 0$

Let $\alpha = .05$

$$r_{XY} = \frac{n\Sigma XY - (\Sigma X)(\Sigma Y)}{\sqrt{n\Sigma X^2 - (\Sigma X)^2} \quad \sqrt{n\Sigma Y^2 - (\Sigma Y)^2}}$$

$$r_{XY} = \frac{10(8855.39) - (270.4)(271.18)}{\sqrt{10(8800.24) - (270.4)^2} \quad \sqrt{10(8912.99) - (271.18)^2}}$$

$$= .9995$$

$r_{XY,df-9,at\ .05} = .521$

As .9995 > .521, reject H_0.

Correlation for H

Category III

Let X ≡ actual data and Y ≡ model prediction

Y values were computed according to $H = \frac{1-q}{q}$

SUBJECT	q	1-q	X	Y
1	.181	.819	8.20	4.52
2	.212	.788	5.33	3.72
3	.450	.550	1.97	1.22
4	.318	.682	4.54	2.14
5	.164	.836	5.84	5.10
6	.164	.836	7.65	5.10
7	.218	.782	5.66	3.58
8	.182	.818	5.02	4.49
9	.221	.779	7.41	3.52
10	.284	.716	4.02	2.52

H_0: $r_{XY} = 0$

H_1: $r_{XY} > 0$

Let $\alpha = .05$

$$r_{XY} = \frac{n\Sigma XY-(\Sigma X)(\Sigma Y)}{\sqrt{n\Sigma X^2-(\Sigma X)^2}\ \sqrt{n\Sigma Y^2-(\Sigma Y)^2}}$$

$$r_{XY} = \frac{10(216.83)-(55.64)(35.91)}{\sqrt{10(341.07)-(55.64)^2}\ \sqrt{10(144.07)-(35.91)^2}}$$

$$= .7804$$

$r_{XY,df=9,\text{ at }.05} = .521$

As .7804 > .521, reject H_0.

APPENDIX B

RAW DATA

Category I(a):

	Step	
Subject 1:	1	000011000010001000000000000001
	2	0000100000000000100000000000000000001
	3	0000000001010000000000000001000000100001 0001
	4	010011
	5	1000110101101001101100101110000111
Subject 2:	1	00001
	2	000000000000000011011
	3	00001
	4	00000000000000000100000010000000001
	5	0001111001000000100000000000001000000000 00000001000001000000000001
Subject 3:	1	0000000000000100000000000100000011
	2	0000000000010000000000011
	3	100110000000011
	4	100000110000001
	5	0000000000000010000000001000000000000000 001101000001000000000000001
Subject 4:	1	000001
	2	0000100000010000000010000010000000001000 001
	3	0011000000001
	4	01
	5	0001011000000000000000000010000000000010 01

Category I(a):

	Step	
Subject 5:	1	11
	2	000000000001100001000000000001010000000111000000000000000000001101100101
	3	0000010000000000000000001
	4	000001
	5	010000001
Subject 6:	1	0010101010011
	2	00100101
	3	0111000010001001000000100011000000001
	4	000110000001000101000110010000000000101
	5	0010010000000000100111000100100100000010000000001
Subject 7:	1	100011011101110101000000000001000000100000000001
	2	110101111001011011100000000000000000110000111000000001011000011000000100011001
	3	000110000110010100000000001
	4	00001
	5	1000010000001100000001
Subject 8:	1	00001
	2	0000011
	3	0000010001
	4	000000001000001
	5	00000000000100000000000000100000000010000010000000000000000000001

Category 1(a):

	Step	
Subject 9:	1	00000000000100000000010001
	2	0001000001011000000011
	3	000000100000001
	4	01000000001
	5	0000001
Subject 10:	1	000000000000000101
	2	001000010101
	3	0001
	4	0001

Category I(b):

	Step	
Subject 1:	1	1000001
	2	1110111101010110100001010000100100011000 00101100110001
	3	1011010000101000001001100100010011011011 00000000000001001
	4	0000000100000100010011111011000101000000 0100000010000000011010000111
	5	01010000100100000000111000000000000011
Subject 2:	1	0000100000011011010000100100010000001000 0000000000100000000000100000100000010000 101001001000000000001
	2	00100001000001
	3	0010011
	4	0001000000000001
	5	0000010000000000010001100000001
Subject 3:	1	1000001000010001000000101000001011000001 0000100000000010001000010001000011100000 0100001
	2	10000010001001000001001
	3	1000010000000001010010100000000000101001 10100010000001110000100000001
	4	0000000100000001
	5	0100000000100010000000010000000000000011 010100000001100000001
Subject 4:	1	0100000011
	2	1100001011011111110100010111000010001001 1011100000001
	3	0000010010000010000001001011001000100000 0000000010000000000010001010100001110011
	4	0000000000101

Category I(b):

	Step	
Subject 4:	5	0000000000101
Subject 5:	1	00010010000000000000000001
	2	101
	3	000010000000110000000000010000000001101100000100000000001
	4	01001000100000100001010000000100001000000000100000000000001101100000001110100000 0100001
	5	010110100000111110001000010001001000000010000001
Subject 6:	1	0100000000000000100101000000000000000101000000000000000000001
	2	00100010000000001000001
	3	00100001
	4	000000001000000000100001001011000000100001
	5	001
Subject 7:	1	000000000000000001
	2	01
	3	00000001
	4	001
	5	0001
Subject 8:	1	00010000000000001000000000000000000010000000000000000000110000100100000000000000001
	2	0000100111010011000010100011
	3	00100111000100010000000000000001111000010111
	4	011

Category 1(b):

	Step	
Subject 8:	5	00000010011011100111110110010101
Subject 9:	1	111000000011
	2	0111010000000001
	3	000010000110010001
	4	000000000000101000000000001
	5	00000001
Subject 10:	1	0010100000000000000000001000100000001000000 0000001
	2	0000000001000010000000000000001001000000 0111110000000000010000000000000100000000 0000010000000000000010000100000000000000 0010000000000001
	3	0101
	4	0000000001001000000000000000010000000000 0000000110011000011110000000000000000000 101
	5	0000010000001100001

Category II:

	Step	
Subject 1:	1	110101111010000010000011000000001
	2	00100000100000000001
	3	01
	4	10011
	5	01
Subject 2:	1	00011
	2	000001001000101001000100101
Subject 3:	1	0000010001111001
	2	000010100011000000000011000001000000010000001000001
	3	000000000001000100010101001001000100001
Subject 4:	1	1101
	2	10101
	3	00000001001
Subject 5:	1	000110000010000000000001
	2	00100000000001000000000000000000010001
	3	00000001
Subject 6:	1	00100000000110000000000000010000000011011
	2	00000000000001
	3	110000000000100001

Category II:

	Step	
Subject 7:	1	001
	2	111111000001
Subject 8:	1	000010000000010001
	2	0000100000100000010111000000001000100001
	3	100000001000000000000001
Subject 9:	1	00001
	2	01100000010000000000101100000000000010001110000000100000000001001000100001000001
Subject 10:	1	0000000000000000000110000011010100000000010000000000100000000001
	2	00100100000000000101
	3	001

Category III:

	Step	
Subject 1:	1	111000000000000010000000010001100001010001
	2	001
	3	110001000000000000000100001
	4	0111000000100010001
	5	0011001000000010000000010001000000000000000000100100000000000000010000000100000000010000001
Subject 2:	1	1000110001000010000000000010000001100100000000000001
	2	010001000000001
	3	0000000000000010000000000000010000000100001
	4	000000000010000000000000000100000000000000001000000000000001
	5	1111111110110000001101001000000000001010110000000100011011101111100100001000100001000001000011100001001
Subject 3:	1	000010000010000110100000000010000000001
	2	01000011
	3	000100001110110000001
	4	111110111010000000010000000001
	5	00001
Subject 4:	1	000000000000100100001000000011001001000000000000000100000001
	2	000000000000100000001
	3	10010000001000000001
	4	000011
	5	0000100001

Category III:

	Step	
Subject 5:	1	00000100000000000001010000001
	2	000001010001
	3	000000000000000011
	4	00000101000000010000100100000000000001
	5	11010000000000010000000001
Subject 6:	1	1000000011100110101000100011111
	2	0000000010000000010000000100000000000000 1000000001
	3	010000010101000000000000000101
	4	000000000100011000000001
	5	1000000000111
Subject 7:	1	000000000000000001
	2	01
	3	000000000000001
	4	00001000001101111
	5	010000001000000000001
Subject 8:	1	00000100000000000000001
	2	0001000000000001
	3	0001001
	4	1000101
	5	1011000001010000110110110000100000100000 1000000100000000101100110000100100000000 000000001000100010000000001001101000

Category III:

	Step	
Subject 9:	1	0111001
	2	10000010000001001111100001011100101
	3	001
	4	111000101
	5	0010101
Subject 10:	1	00000001
	2	111101001
	3	1001000101000000000100100111010101000001 1
	4	1100000000000010000000001
	5	011

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